Detection of nanocylinder dimensions using sequential spatial mode projection

Nestor D. Bareza Jr.* **and Nathaniel Hermosa**

National Institute of Physics, University of the Philippines Diliman, Quezon City, Diliman 1101

*Corresponding author: nbareza@nip.upd.edu.ph

Abstract

We present a dimensional assessment of a nanocylinder through spatial mode projection. We analytically solve the output power of the beam that is incident to a nanocylinder and whose reflection is projected onto various modes. Among the modes that we try, the Gaussian mode and the tailored cylindrical mode produce power expression *P's*, that can be related to the height and the radius. The Laguerre-gausian beams on the other hand, either do not show any expression that has a relation to the dimensions, or the factors containing these dimensions have negligible contributions. However, the *P's* obtained for both Gaussian mode and the tailored cylindrical mode have inseparable factors that contain the information about the height and the radius. We therefore propose that a sequential projection of the Gaussian mode and the tailored mode to explicitly determine the height and the radius.

Keywords: Laser optical systems: design and phase disturbance of operation 42.60.-v,and Laser applications 42.62.-b

1. Introduction

The equipment development for nanoscale characterizations are of great interest ever since the birth of nanotechnology. As the materials science approached toward nanoscale, new research grounds emerged in exploring precise control and detection of nanoscale features. The morphological characteristics and arrangements of nanoparticles could be studied through scanning-probe method, x-ray diffractometer and electron microscopy [1–3].

Optical characterizations have been a useful substitute for other highly sophisticated facilities in observing nanofeatures due to its easier implementation and low-cost maintenance. A technique called fluorescence correlation spectroscopy is being used for particle or quantum dot tracking in a brownian motion [4]. A recent work used spatial mode projection as a technique in measuring a nanostep height smaller than 10 nm [6]. In their work, the reflected beam from a nanostep was projected to a tailored spatial mode before measuring the output power. The result demonstrated a linear relationship between power detected versus tiny height changes.

This work, an extension of [6], presents the first study of the use of mode projection on characterizing a two-dimensional nanofeature. Illuminating a certain laser mode on a nanofeature introduces a phase shift on the spatially varying phase of the beam. An information could be extracted by detecting the output intensity of the reflected beam [5]. We are particularly interested in obtaining the radius r_c and the height h of a nanocylinder. This technique requires that the shape should be priorly known before selecting an appropriate mode.

Using a Gaussian mode $G(r, \phi)$ as incident beam, the nanocylinder with a height h would induce a phase shift $\tau(r, \phi)$ upon reflection. $\tau(r, \phi)$ takes the form

$$\tau(r,\phi) = e^{i\delta};\ \delta = \begin{cases} \frac{2\pi}{\lambda}2h \text{ for r} \leq r_c \\ 0 \text{ for } r > r_c \end{cases} \tag{1}$$

where λ is the wavelength of the laser beam. Projecting an appropriate mode $M(r, \phi)$ to the reflected beam, the output power can be calculated given by the following integral in the polar coordinates [8]:

$$P = \left| \int_0^{2\pi} \int_0^{\infty} G(r,\phi)\tau(r,\phi)M(r,\phi) r dr d\phi \right|^2 \tag{2}$$

We solve Equation 2 to determine the most appropriate mode to use in determining the dimension of the nanocylinder.

2. Methodology

Equation 2 is based on the experimental setup illustrated in Figure 1. In this scheme, the nanocylinder situated on a substrate is being illuminated by a beam. For simplicity, the beam is chosen to be a Gaussian mode. The reflected beam is projected to a chosen mode and the power is obtained. We projected the reflected beam onto different modes which are used in calculating Equation 2.

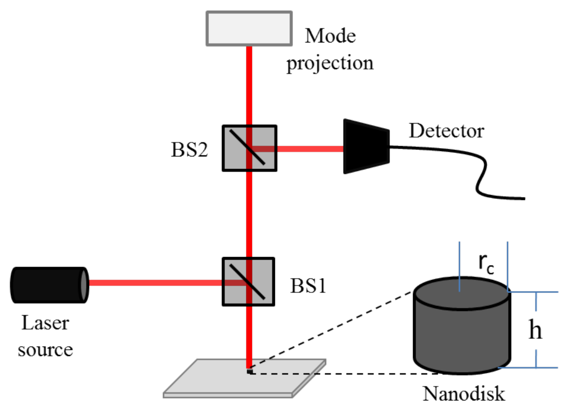

Figure 1: Illustration of an experimental setup for a mode initially projected to the reflected beam from a nanocylinder before the output power is being detected.

We consider projecting the reflected beam onto modes that have the same cylindrical symmetry in order to seek the relationship between power and nanocylinder dimensions. First, we use a Gaussian mode which is a radially symmetric beam. Second, we projected the reflected beam to a Laguerre-gauss mode. This mode is cylindrically symmetric and has a phase with an azimuthal angular dependence ($\exp(-il\phi)$) along propagation [7]. Lastly, a mode is tailored to be identically a cylinder with radius r_m ($> r_c$) that introduces a phase-shift $\Delta\varphi$ to the reflected beam. As a summary, the modes considered for calculation are explicitly written as follows:

$$\text{Gaussian mode: } M_1 \sim \exp\left(-\frac{r^2}{\omega_o^2}\right) \tag{3}$$

$$\text{Laguerre-Gauss mode: } M_2 = LG_{l,p} \sim r^l L_p^l\left(\frac{2r^2}{\omega_o^2}\right)\exp\left(-\frac{r^2}{\omega_o^2}\right)\exp\left(-il\phi\right) \tag{4}$$

where L_p^l is the generalized Laguerre polynomial

$$\text{Tailored mode: } M_3 \sim \exp\left(-\frac{r^2}{\omega_o^2}\right)\exp\left(\Delta\varphi'\right);\ \Delta\varphi' = \begin{cases}\Delta\varphi; r \leq r_m\\ 0; r > r_m\end{cases} \tag{5}$$

3. Results and Discussion

Using Gaussian mode for projection, the resulting power without the nanocylinder or directly projecting a Gaussian mode to the undisturbed beam in Gaussian mode is unity. The presence of nanocylinder introduces a phase disturbance to the Gaussian beam thus we expect a resulting non-unity value of power. The resulting power, denoted by P_1 after projection of the reflected beam from a nanocylinder is given by

$$P_1 = 1 - \left(\frac{r_c}{\omega_o}\right)^2 \delta^2;\ \omega_o > r_c \tag{6}$$

Notice that it has a subtrahend that has inseparable factors of radius and height.

In the expression acquired for P_1, the expansions for the exponential and cosine functions that appeared were taken only up to the second order. This is valid since the radius and height of nanocylinder is significantly smaller than the beam waist ω_o and wavelength, respectively. These approximations were applied for the succeeding calculations in this paper.

The integral forms of radial and angular parts could be separately evaluated as Laguerre-gauss mode appear in the integrand of Equation 2. The azimuthal angular dependence acts as integrand of the angular part and the case for l is not equal to zero was first investigated. Its definite integral when evaluated from zero to 2π always results to zero hence, the resulting power becomes zero. Generally, there is no output power to be detected for projecting Laguerre-Gauss mode with an azimuthal angular dependence. For the second case (l is equal to zero and p is not zero), the resulting power is almost negligible since the highest order of the ratio between r_c and w_o that appeared is in the fourth order. An exceptional case is when both l and p is equal to zero; the power calculated is equal to P_1 since the Laguerre-gauss with this indices is also a Gaussian mode.

$$P_2 = \begin{cases} 0 \text{ for } l > 0 \\ \sim 0 \text{ for } l = 1 \text{ but } p > 0 \\ P_1 \text{ for } (l, p) = (0, 0) \end{cases} \tag{7}$$

For the tailored mode, the resulting power denoted by P_3 does not only depend on dimensions of nanocylinder but also to parameters of the tailored mode.

$$P_3 = 1 - 4\left(\frac{r_m}{\omega_o}\right)^2 + 4\cos(\delta + \Delta\varphi)\left(\frac{r_c}{\omega_o}\right)^2 + 4\cos(\Delta\varphi)\left(\left(\frac{r_m}{\omega_o}\right)^2 - \left(\frac{r_c}{\omega_o}\right)^2\right); \omega_o > r_m > r_c \tag{8}$$

Since the dimensional characteristic of tailored mode can be controlled, the value for phase-shift $\Delta\varphi$ can be set to $\pi/2$ so that Equation 8 gives a simpler expression as shown in Equation 9. The value for r_m can be set to a certain value for convenience, however we leave it in general since it can be set as a certain value for convenience.

$$P_3 = 1 - \frac{4}{\omega_o^2}\left(r_m^2 + r_c^2\delta\right); \omega_o > r_m > r_c \tag{9}$$

This expression also manifests inseparable height and radius factors in the subtrahend part similar to the expression of P_1. Both powers P_1 and P_3 are related to the square of nanocylinder radius however they differ in the order of δ. In Equation 6, P_1 is related to second order of δ whereas in Equation 9, it is first order in δ. By using tailored mode M_3, we are able to decrease the order of the inseparable factors hence, increasing the detectable change in the power.

The two expressions of powers, P_1 and P_3, signify that it is inevitable to have a coupled factor of the radius and the height of the nanocylinder in the resulting power. This suggests that in measuring the radius and height of nanocylinder, a sequential spatial mode projection can be performed using a Gaussian mode and a tailored cylindrical mode. By eliminating the radius variable of nanocylinder in Equations 6 and 9, the height can then already be determined as represented by δ in the following equation:

$$\delta = \frac{4(1 - P_1)}{1 - P_3 - 4\left(\frac{r_m}{\omega_o}\right)^2} \tag{10}$$

Moreover, the radius of nanocylinder could be extracted by substituting the δ value of Equation 10 back to the expression of either Equation 6 or 9 which is derived to be:

$$r_c = \frac{\omega_o}{4}\frac{1 - P_3 - 4\left(\frac{r_m}{\omega_o}\right)^2}{\sqrt{1 - P_1}} \tag{11}$$

In actual experiment, $\delta^2\left(\frac{r_c}{\omega_o}\right)^2 \approx 10^{-4}$ and hence, this small change can be embedded in the random noise. Some techniques to reduce technical noise may be needed. The spatial mode projection technique had shown that a resolution of fraction in nanometers could be detected with a nanostep feature as presented in [6]. In this work, it is expected that a lower order of magnitude of resolution could be obtained since second order of δ is derived in Equation 9. However, the resolution will still depend on sensitivity of the equipment detection, suppression of noise, and stability of experimental setup.

4. Conclusion

In summary, the two dimensions that describe the nanocylinder can be measured through sequential spatial mode projection as demonstrated by analytical approach in this study. The information about the radius and the height was found to be embedded as coupled factors in the calculated powers. In order to resolve this, both the values of radius and height could be explicitly determined by sequentially projecting a Gaussian mode and a tailored mode to the reflected beam. In actual experiment, the tailored mode could be easily modified using controllable phase elements such as spatial light modulator.

In a previous study, a single mode projected to reflected beam from a nanostep can already identify the height measurement between layers [6]. In this study, two modes are required for spatial mode projection in able to measure the two dimensions of a nanocylinder. From these two findings, we present the following conjecture: In order to extract the desired information of a nanofeature, the number of dimensions that contribute to phase disturbance of the illuminated beam should be taken into account; This is the same number of choosing appropriate modes in explicitly acquiring the dimension values. Finally, we propose that it might also be possible to use two strongly coupled polarization and spatial modes as incident beam in detecting dimensions of the nanocylinder.

References

[1] Nishijima, Hidehiro, et al. "Carbon-nanotube tips for scanning probe microscopy: Preparation by a controlled process and observation of deoxyribonucleic acid, "*Applied Physics Letters* 74.26 (1999): 4061-4063.

[2] Zhang, X. F., et al. "Carbon nano-tubes; their formation process and observation by electron microscopy, "*Journal of crystal growth* 130.3 (1993): 368-382.

[3] Waddon, Alan J., and E. Bryan Coughlin. "Crystal structure of polyhedral oligomeric silsequioxane (POSS) nano-materials: a study by X-ray diffraction and electron microscopy, "*Chemistry of materials* 15.24 (2003): 4555-4561.

[4] McHale, Kevin, Andrew J. Berglund, and Hideo Mabuchi. "Quantum dot photon statistics measured by three-dimensional particle tracking "*Nano letters* 7.11 (2007): 3535-3539.

[5] Schuster, R., et al. "Phase measurement in a quantum dot via a double-slit interference experiment, "*Nature* 385.6615 (1997): 417-420.

[6] Hermosa, Nathaniel, et al. "Nanostep height measurement via spatial mode projection, "*Optics letters* 39.2 (2014): 299-302.

[7] Allen, Les, et al. "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes, "*Physical Review A* 45.11 (1992): 8185.

[8] Molina-Terriza, Gabriel, et al. "Probing canonical geometrical objects by digital spiral imaging."*Journal of the European Optical Society-Rapid Publications* 2 (2007).