Bessel-like beams with controllable path

Jobenn Sabanal^{1*}, Mayvilyn Bondoc¹, and Nathaniel Hermosa²

¹Department of Physical Sciences, College of Science, Polytechnic University of the Philippines, Sta. Mesa, Manila 1016 Philippines

²National Institute of Physics, University of the Philippines Diliman, Quezon City 1101, Philippines *Corresponding author: sjobenn@gmail.com

Abstract

In this paper, we present two methods to control the propagation path of a Bessel-like beam. We show that: 1) by adding a carrier frequency f, the beam can be made to deflect with a constant angle; and 2) a second order modification in the conical phase makes the beam deflect akin to an accelerating beam. We observe that both beams with modified phases self-reconstruct. We note however, that the second method of controlling deflection needs to be characterized in more detail since the beam seems to lose its propagation invariant property.

Keywords: 42.25.Fx (Diffraction, optical); 42.65.Jx (Phase); 42.15.Dp (Wave fronts and ray tracing)

1. Introduction

There has been a huge interest in the study of nondiffracting and self-reconstructing beams since the seminal paper of Durnin in 1987. This is mainly buoyed by the range of applications such as in imaging through scattering media [1], in optical manipulation [2,3], and in light bullet production [4] among others. A nondiffracting optical beam propagates in free space without spreading while a self-reconstructing beam is hardly affected by a small block placed along its path [5–9]. The self-reconstructing and nondiffracting properties are not mutually exclusive: a beam can be nondiffracting but not self-reconstructing, and vice versa [7–9].

Nondiffracting and self-reconstructing beams can be accelerating or non-accelerating. Accelerating nondiffracting beams tend to move along a curved path [10–13], as opposed to non-accelerating nondiffracting beams that propagate in a straight path [14–16]. One of the most known examples of nondiffracting, self-reconstructing and non-accelerating beams is the Bessel beam [14].

Bessel beam, which has a conical phase, can be produced in a variety of method: they can be generated with the use of an axicon or with a computer generated hologram. An axicon is a cone made of dielectric material that gives a conical phase to a beam that passes through it while a computer generated hologram imprints that phase to a reconstructed beam. The conical phase of a Bessel beam can be written as

$$\psi(x,y) = 2\pi\alpha \left(1 - \frac{\sqrt{x^2 + y^2}}{r_0}\right) \tag{1}$$

where x and y are the coordinates, r_0 is the aperture radius, and α gives the slope of the phase gradient.

In this study, we propose a method to generate Bessel-like beams with controllable propagation path by modifying equation 1. We want to produce a beam that has the nondiffracting and self-reconstructing properties of a Bessel beam yet we have the flexibility to change the beam's path as we please for various applications. We do this by adding 1) a carrier frequency and 2) a nonlinear modification in the cone factor. The addition of carrier frequency introduces a tilt to the phase. While the nonlinear modification on the conical phase introduces a second order aberration to the system. We note that there have been studies on Bessel beam with curved trajectories before [17,18]. Our method, however, is far more simple than the optimization with constraints methods used in those papers. We want to simplify a way in generating accelerating Bessel-like beams using our method.

2. Methodology

The split-step algorithm, which utilized angular spectrum decomposition, is used to simulate the free space propagation of the beam. With this algorithm, the electric field E(x, y, z) at different propagation distance z can be obtained. The initial field used in our simulation is of the form,

$$E(x, y, 0) = E_0 exp(-i\psi(x, y))$$
⁽²⁾

where E_0 is taken as unity and the phases $\psi's$ are given by eqns. 3 and 4.

$$\psi_1 = 2\pi\alpha \left(1 - \frac{\sqrt{x^2 + y^2}}{r_0}\right) + 2\pi f(x+y)$$
(3)

$$\psi_2 = 2\pi\alpha \left(1 - \frac{\sqrt{(x - ax^2)^2 + y^2}}{r_0}\right) \tag{4}$$

where f is the carrier frequency and a is a free parameter that changes the opening of the quadratic function. In eqn 3 the second term has the added carrier frequency. In our simulations, the carrier frequency is added in the x coordinate only. In eqn 4, a quadratic function is subtracted to x.

The beam's position was obtained as the position of its maximum intensity. We also checked if the beams reconstruct after they have been blocked by a small obstacle using the same algorithm.

3. Results and Discussion

When the conical phase of a Bessel beam is modified, rays can be refracted at constant or different angle.



Figure 1: $\psi_1(a) 2\pi$ -modulo phase, propagation profile with f = 4/mm at (b) z=0.1 m (c) z=0.3 m (d) z=0.5 m, (e) axial profile at different propagation distance z for f = 4/mm.

Upon adding a carrier frequency f, the phase profile ψ_1 is shown in fig 1(a) produced a Bessel-like beam. This beam possesses a nondiffracting property as observed in fig 1(b-d). Notice that the beam deflects at constant angle along the x-axis. The f is added only on the x component of the phase, hence its behavior. The deflection is linear as shown in fig 1(e). This is expected as an f is usually added to computer-generated holograms to separate beams (see for example [7]). Although it is not shown in the figure, we tried different f's and observed that the deflection increases with it. The f acts as a grating that linearly deflects beams. Since f can be controlled, it follows that the deflection of the beam can also be controlled.

The addition of carrier frequency introduces a tilt to the phase, resulting to the deflection of the Poynting vector in the direction of the tilted phase. Hence, the beam propagates in the tilted direction. The addition of f caused the beam to deflect. This is due to the refraction of wavefront at different angles. As expected, waves were superimposed relative to the direction of the tilted phase.

The phase profile ψ_2 is depicted in fig 2(a). In this beam, the phase is nonlinearly modified by subtracting an ax^2 factor in the conical expression. By doing this, the beam deflects as shown in fig 2(b-d). Its deflection as a function of distance z is nonlinear based on fig 2(e). This is a behavior of an accelerating beam. The opening of the nonlinear deflection of the beam is set by the free parameter a. The deflections at different a's collapse to a quadratic equation with $1/a^2$. As we observed, the larger the free parameter a, the larger the deflection of the beam. In our simulation for example, a deflection of 20x the beam diameter happened at z=0.6 m for a = 200. This can still be increased with higher a values. But this will be in expense of the beam profile.

Although we have nonlinearly modified the phase, we made sure that the modification is very small so that the cone will not be altered drastically. ax^2 in our simulation is just 10 percent of x. This values is just enough to say that the conical property of the beam was not destroyed.



Figure 2: $\psi_2(a) 2\pi$ -modulo phase, propagation profile of a = 100 at (b) z=0.1 m (c) z=0.3 m (d) z=0.5 m, (e) axial profile at different propagation distance z for a = 200.

Unlike the beams that are linearly deflected, the profile of the beam with ψ_2 losses its propagation invariant property as shown in fig 2(d). We ascribed this to the phase that losses its symmetry due to the addition of the ax^2 factor.

Even though the beam losses its invariant property, the center spot however, is still visible up to a propagation distance of 0.6 meters. According to Durnin in [14], a beam still behaves a nondiffracting property if the center spot of the beam was retained after propagation.

Our simulation shows that the beam in figure 2 is indeed nondiffracting, for it retained its property after propagation similar to a nondiffracting property of a Bessel beam.



Figure 3: Reconstruction profile at different z (units in meter) of (a) ψ_1 , (b) ψ_2 .

A block with dimensions 0.0004m x 0.0004m is placed at the beam's path. The path deflection of the beam when blocked is shown in fig 3. This shows that both beams with modified phases selfreconstruct. The propagation profile of both beams before and after blocking seems to be similar. However, reconstruction caused the intensity of the beam to decreased upon propagation. We observed that when the beam is blocked, the deflection along the x coordinate is larger than when the beam is not blocked. This is an interesting result which might be answered by checking the internal energy of the beam as it propagates. This however, is an independent study which will be explored soon and not part of this paper.

We note that the electromagnetic field with the modified phase ψ_2 needs to be characterized in more detail since the beam seems to lose its propagation invariant property as we described above.

4. Conclusion and Recommendation

This paper presented two methods to control the propagation path of a Bessel-like beam. It is observed that by adding a carrier frequency f to the conical phase, the beam can be made to deflect at constant

angle. As expected, the deflection of the beam is linear. The second order modification on the conical phase caused the beam to accelerate: its deflection is not proportional to the propagation distance z. We further observed that both beams with modified phases self reconstruct. We intend to study the internal energy of the beam as it propagates to understand the behavior of the beam when blocked.

References

- F. O. Fahrbach and A. Rohrbach, "Improved Bessel beam light sheets," Nat. Commun. 3, 632 (2012).
- [2] V. Garcés-Chávez, D. McGloin, H. Melville, W. Sibbett, and K. Dholakia, "Simultaneous micromanipulation in multiple planes using a self-reconstructing light beam," Nature 419, 145-147 (2002).
- [3] J. Baumgartl, M. Mazilu, and K. Dholakia, "Optically mediated particle clearing using Airy wavepackets," Nat.Photon. 2, 675 - 678 (2008).
- [4] A. Chong, W. H. Renninger, D.N. Christodoulides and F. W. Wise, "AiryBessel wave packets as versatile linear light bullets," Nat. Photon. 4 103 (2010).
- [5] J. Broky, G. Siviloglou, A. Dogariu, and D. Christodoulides, "Self-healing properties of optical Airy beams," Opt. Express 16, 12880-12891 (2008).
- [6] J. Ring, J. Lindberg, A. Mourka, M. Mazilu, K. Dholakia, and M. Dennis, "Auto-focusing and self-healing of Pearcey beams," Opt. Express 20, 18955-18966 (2012).
- [7] N. Hermosa, C. Rosales-Guzmán and J.P. Torres, "Helico-conical optical beams self-heal," Opt. Lett. 38, 383-385 (2013).
- [8] P. Vaity and R. P. Singh, "Self-healing property of optical ring lattice," Opt. Lett. 36, 2994-2996 (2011).
- [9] M. Anguiano-Morales, A. Martínez, S. Chávez-Cerda, and N. Alcalá-Ochoa, "Self-healing property of a caustic optical beam," Appl. Opt. 46, 8284-8290 (2007).
- [10] G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, "Observation of Accelerating Airy Beams," Phys. Rev. Lett. 99, 213901 (2007).
- [11] E. Greenfield, M. Segev, W. Walasik, and O. Raz, "Accelerating Light Beams along Arbitrary Convex Trajectories," Phys. Rev. Lett. 106, 213902 (2011).
- [12] M. A. Bandres, "Accelerating beams," Opt. Lett. 34 3791-3793 (2009).
- [13] P. Zhang, Y. Hu, T. Li, D. Cannan, X. Yin, R. Morandotti, Z. Chen, and X. Zhang, "Nonparaxial Mathieu and Weber Accelerating Beams," Phys. Rev. Lett. 109, 193901 (2012).
- [14] J. Durnin, "Exact solutions for nondiffracting beams. I. The scalar theory," J. Opt. Soc. Am. A 4, 651-654 (1987).
- [15] M. A. Bandres, J. C. Gutiúrrez-Vega, and S. Chávez-Cerda, "Parabolic nondiffracting optical wave fields," Opt. Lett. 29, 44-46 (2004).
- [16] S. Chvez-Cerda, M. J. Padgett, I. Allison, G. H. C. New, J. C. Gutiérrez-Vega, A. T. O'Neil, I. MacVicar, and J. Courtial, "Holographic generation and orbital angular momentum of high-order Mathieu beams," J. Opt. B: Quantum Semiclass. Opt. 4 S52 (2002).
- [17] I. D. Chremmos, Z. Chen, D. N. Christodoulides, and N. K. Efremidis, "Bessel-like optical beams with arbitrary trajectories," Opt. Lett. 37, 5003-5005 (2012).
- [18] J. E. Morris, T. imr, H. I. C. Dalgarno, R. F. Marchington, F. J. Gunn-Moore, and K Dholakia, "Realization of curved Bessel beams: propagation around obstructions," J. Opt. 12, 124002 (2010).